AMERICAN SOCIETY OF CIVIL ENGINEERS.

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TRANSACTIONS.

NOTE.—This Society is not responsible, as a body, for the facts and opinions advanced in any of its publications.

CCLIX.

(Vol. XII.—July, 1883.)

AN ACCIDENT TO STEAM PIPES ARISING FROM THE USE OF BLAST FURNACE WOOL.

By T. EGLESTON, Ph. D., M. Am. Soc. C. E.

READ AT THE ANNUAL CONVENTION OF THE SOCIETY, HELD AT ST. PAUL AND MINNEAPOLIS, MINN., JUNE 19TH, 1883.

WITH DISCUSSION

By CHARLES E. EMERY, M. Am. Soc. C. E.

At the Hartford meeting of the Society of Mechanical Engineers, Mr. C. E. Emery, M. Am. Soc. C. E., of New York, read a paper on Experiments with Non-Conductors of Heat.* The results of his investigations with various non-conductors show such a remarkable power of retaining heat in blast furnace wool, that this substance has since that time been very largely employed for this purpose. This is owing to the long fibrous condition of the material, which mats together into an exceedingly light structure capable of retaining so much air that, while itself a good conductor of heat, it becomes eminently a bad one from the

*Transactions of the Society of Mechanical Engineers, Vol. II., p. 34.

very large volume of air contained in it. An explosion which happened at Columbia College has drawn attention to this subject. A paper was read by Prof. F. R. Hutton, of the School of Mines,* calling attention to this accident. As it might have been a serious one, and as a similar accident is likely to happen in almost any place where this substance is used around steam pipes, I have thought it worth while to give to the Society a resumé of the investigations which I made as to its cause for that paper, and some additional ones which I have since made, for the purpose of calling the attention of this Society to a series of important facts in relation to the use of this substance.

The buildings of Columbia College are heated by steam pipes which are encased in zinc, the interval between the pipe and the zinc outside being filled with the blast furnace wool, packed into the casing. Two pipes situated in different positions burst. One was 5 inches in diameter, and carried steam at a pressure of 40 pounds. It was in a sheathing, with two others, packed with the mineral wool. These pipes were supported on hangers. The moisture came from joints rendered leaky by decomposition of the rubber packing, and from the motion caused by the contraction and expansion at the joint. The sheathing was properly packed. The first effect of the moisture was to cause a sagging of the wool to the lower part of the metal sheathing, causing the top of the pipe to become uncovered, and making a concentration of the moist material around the lower part of the pipe. The threads of this pipe were completely obliterated. The other pipe was 2½ inches in diameter, and was laid in a brick conduit covered with flagstones. This conduit was open at both ends, and any moisture should have found an exit, but owing to the sagging of the mineral wool it failed to do so, but settled around the lower part of the pipe, which was attacked by it until it became so thin that it blew out in about the middle of the pipe. A 3-inch pipe, belonging to the same system, laid in the ground, without any covering to protect it from the earth, which was dug up about the same time after having been buried seven and a half years, was found to be perfectly sound. Both of these small pipes have been subjected to exactly the same conditions, as far as the steam was concerned. They were carrying the condensed steam back to the boilers. Seeing at once what the cause of the accident was, I made a careful examination of the conditions, ascertaining beyond a doubt that the cause

* Transactions of the Society of Mechanical Engineers, Vol. III., p. 228.

of the corrosion was the action of the sulphuric acid set free from the packing.

The mineral wool was prepared from a slag, whose analysis is given below:

Water	1.08
Potash	0.19
Soda	1.75
Lime	26.56
Magnesia	19.82
Alumina	7.84
Sesquioxide of Iron	0.64
Sulphur	2.46
Silica	38.97
	99.31

Some of this slag was soluble, and some of it insoluble in water, as is shown by the analysis below:

SOLUBLE.		INSOLUBLE.	
Water	1.08	Alumina	7.84
Potash	0.19	Sesquioxide of Iron	0.64
Soda	1.75	Silica	38.97
Magnesia	0.12	Magnesia	19.70
Lime	1.61	Lime	24.95
Sulphur	0.23	Sulphur	2.23
	4.98		94.33

There was, therefore, a certain amount of acid material ready to act on the pipes the moment sufficient moisture was present to dissolve it, which was, however, quite harmless while the materials were dry.

The blast furnace slag from which the mineral wool is made, is not a material of constant composition. It varies with the different kinds of ore and fuel used, and the changes in the working of the furnace from day to day, the conditions rarely being exactly the same for any great length of time. The addition of lime in considerable quantities to the charge, is the only method known to metallurgists for separating the sulphur introduced with the ore into the furnace, for which purpose, when sulphur is present in the ore, it is usual to make the furnace carry as large a burden of flux as is possible. When the ore contains no sul-

phur the slag will contain only that contained in the flux and fuel; some slags contain very little and some none, and such should always be selected for the manufacture of the wool; such slags are, however, rare and are seldom produced in localities where the manufacture of the wool would be profitable. Independently of the quantity of lime, however, all the blast furnace slags have certain characteristics in which they are alike. They usually, when they contain silica enough, gelatinize with acids. When, however, they are very basic, they fall to pieces on exposure to the air. This gelatinization with acids not only takes place with the ordinary mineral acids, but also with the ordinary organic acids. Very weak solutions of oxalic, tartaric, citric and acetic acids effect the decomposition, and it is likely to be effected in acid slags by any of the acids or alkalies found in the soil when there is any drainage passing through it. If the slag is very basic, any water will decompose it. It is not necessary, however, that the complete decomposition should take place in order to set the sulphuric acid free. The mineral wool is in such a very fine state of division that, coming in contact with moisture alone, the soluble part would not only be quickly dissolved out, but a commencement of decomposition would be likely to take place. This is shown by the fact that blast furnace slag in a granulated condition is used in France and Germany* to a very large extent for sand for mortar. It is very extensively used in many parts of Europe, when mixed with a small quantity of lime for making sun-dried bricks, which, as the bricks are large, make very cheap building material, which in a very short time by the chemical action becomes quite hard, and as they are light and porous are both better and cheaper than brick. It is also, owing to the peculiar condition of the silica, extensively used in some parts of Germany for the manufacture of Portland cement, and in many parts of Europe very largely as a fertilizer, for all of which purposes the property of decomposing and forming a new chemical compound when exposed to the air and moisture is relied on. In every case, the decomposition which is effected to make another chemical union, as in the case of mortar, sand, bricks and cement, or for the purpose of permitting the easy absorption of some of its constituents into the sap of plants, as in the case of its use as a fertilizer, sets the sulphur which it contains free to act on whatever it finds to act upon, which, in this case, was the iron. The analyses given below, made in my laboratory and

* Trans. Amer. Inst. Mining Engineers, Vol. I., p. 212.

taken at random from my note-book, show that sulphur is contained in all slags, and although the slag wool, which caused the accident, contained more than most of those cited, as they are from furnaces whose ores contain but little sulphur, the accident is likely to happen wherever the slag wool is used:

	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.
Silica	37.000	36.50	38.52	38.92	40.20	38.62	35.50	38.52
Alumina	15.535	8.45	8.92	9.96	10.08	9.89	8.45	8.92
Lime	40.265	47.92	47.43	45.85	44.89	40.21	47.92	47.43
Magnesia	3.790	1.62	1.73	1.32	1.74	3.79	1.62	1.73
Sesquioxide of iron	0.156	3.02	1.16	2.38	1.15	1.22	3.02	1.16
" manganese	0.004	1.91	0.634	1.05	0.77	0.72	1.91	0.634
Phosphoric acid	.....	0.15	0.105	.....	0.03	0.18	0.15	0.105
Sulphur	1.332	1.10	1.086	1.02	1.04	3.07	1.10	1.086
	98.082	99.67	99.585	100.50	100.90	97.70	99.67	99.585

No. 1, Fletcher Furnace, Buffalo, N. Y.
No. 2, 3, Ferrol, Va.
Nos. 4, 5, 6, 7, 8, Quinnemont, W. Va.

That the sulphur is set free and does act is shown by the analysis of the scale removed from the pipes at the point where they gave way, which could be easily detached with the nail in pieces of considerable size.

ANALYSIS OF THE SCALE.

Lime	3.09	Silica	2.46	
Magnesia	1.07	Water	4.38	
Protoxide of iron	3.98	Sulphuric acid	3.75	
Sesquioxide of iron	81.51			100.24

This analysis shows a considerable quantity of sulphuric acid still remaining in the scale, although it had been exposed for some time to the air and moisture before it was brought to me to be examined, under which conditions the iron would generally be thrown down in the shape of sesquioxide while the sulphuric acid would be liberated and dissolved out.

There is likely to be a considerable amount of variation in the temperature around pipes in ordinary manufactories, except where they run

night and day, and there will be always a probability of having moisture come into contact with the slag, which may come either from the condensation from the occasional lowering of temperature, or from leakages of the pipes, or from snow or rain. When the pipes are suspended in the air they are subject to the influence of these kinds of moisture alone. When, however, they are buried in the soil, they are affected not only by these, but by many substances that may be dissolved by the water which percolates through the earth in which the pipes are buried. It is a well-known fact that healthy earth, not subject to any drainage, contains a certain amount of both alkaline and acid materials, and that this is sufficient in many cases to cause decomposition even in the rocks lying in contact with them. In order to ascertain exactly what the effect of drainage waters would be likely to be, I made the following series of experiments. A portion of the slag was added to solutions of citric, acetic, tartaric and oxalic acids, each solution being just strong enough to give a decided red reaction with litmus paper, and left for an hour. At the end of that time each solution showed the silica in a gelatinous condition, and the sulphur, therefore, disengaged and ready to attack any substance with which it came in contact. One gramme of the material was then taken and boiled for five hours in 100 cubic centimeters of the following solutions, with the following results:

Reagents Used.	Residue.	Sulphur in Solution.
Distilled Water	97.02	0.23
Croton "	97.69	0.40
" " with Tartaric Acid	87.65	0.52
" " " Citric "	85.58	0.39
" " " Oxalic "	94.58	0.45
" " " Acetic "	84.56	0.39
" " " one per cent. Salt	96.72	0.50
" " " Carbonate of Ammonia	96 22	0.41
" " " " " Potash	97.72	0.64
" " " " " Soda	99.06	0.56

These solutions were rendered so slightly alkaline or so slightly acid that they would just affect litmus paper. It appears from this that while distilled water dissolved only a very small amount of the sulphur present, nearly every one of the other reagents, although they were so very dilute, caused more than double the amount of the sulphur in the slag to go into solution, than with pure water, and that this, while it was at a minimum with the acids, rose to a maximum with the alkalies; so that in the case of those salts most likely to be found in the soil, the amount of sulphur dissolved was, in the case of the potash salt, more than three times the amount dissolved by pure water, and that the alkalies were generally more powerful than the acids. In making these experiments every effort was made to find an alkaline or an acid solution so weak as not to cause a decomposition of the wool without the aid of heat. This, however, was found to be quite impossible, for although a considerable portion of the slag remained unattacked, some of the silica was always gelatinized and could be distinctly seen in that condition. As salt is one of the constant constituents of drainage waters, and as both the alkaline and the acid materials, more especially salt, are likely to be found in the waters draining through the soil of cities, these results are extremely interesting as showing the ease with which this substance may be attacked.

In order to ascertain whether sulphur was likely to be separated from the slag under ordinary conditions, a given weight of the wool was heated to below dull red in a tube, and over this a current of dry hot air was rapidly passed for some time. No separation of the sulphur took place, and none was given off, showing that this substance is quite safe so long as the material is kept dry. The wool was then heated in a tube as before, and cold air, which had been aspirated through a considerable depth of water, was passed over it, but no decomposition of the slag and no separation of sulphur took place, showing that slightly moist air is not dangerous. It was intended to have made the experiment of passing steam over the wool, but before apparatus for doing it could be got ready the fires in the boilers of the steam-heating apparatus were shut down, and this experiment was not made. It is, however, of less importance, as it is certain from the corrosion, which caused the explosion, that the slag wool would have been decomposed.

The slag transformed into wool does not differ in any respect from the slag in a solid condition, except from the fact that its fibres become interwoven. It occupies, when not compressed, a maximum volume for

a minimum weight of the slag, and thus retains a very large quantity of air; and this is its only value. It is the air and not the slag which is required. The only value of the slag is its capability of holding this air when it is not compressed. When it is compressed it has very little more value than the slag in its solid form. This compression may be caused by the sagging of the pipes on it when only the envelope and not the pipe is supported, or by its becoming soaked with water when it mats together, and water takes the place of the air, when it ceases to be a non-conductor of any value. Its great value is in its very fine fibres; but it is precisely this quality of fine division which makes it most dangerous, for in this condition it is most easily attacked by organic acids, by alkalies or by moisture, which not only decompose it, but render the pipes, as in this case, liable to attack. Even the commencement of decomposition causes it to sag and settle.

It would seem, therefore, that the mineral wool, if made from slags containing sulphur, is, under certain conditions, a dangerous material. In one of the cases mentioned the moisture undoubtedly came from defective joints, which are likely to belong to any other system. In the other and far more dangerous one, this moisture was that of condensation, and as it was not expected, as every precaution was taken against it, it is by far the most dangerous one, as it would not be looked for, while every joint would from time to time be visited. In any system moisture is likely to come from rain or snow leaking through the envelope, where the pipes are exposed above ground in the open air, or, when they are below ground, from drainage water, and, in both cases, from condensation, from sudden cooling or too sudden heating of the pipes. The effects produced are likely to be all the greater in intensity as the pipes are hotter, as this facilitates the liberation of the sulphuric acid, which produces a further decomposition of the slag and keeps setting free new portions of acid to further decompose the slag and to attack the iron. The moment the silica commences to assume the gelatinous condition the other constituents of the slag are set free to attack the pipes, and unless the leakage is found and stopped an accident is sure to happen. It seems, therefore, a wise precaution, when this substance is used, to use it only where leakage is not likely to occur or moisture to collect, and to carry the whole system above ground, under cover; or, when it is necessary to carry it below the surface, to have all parts of it easily accessible, so that it may be carefully examined from time to time. Beneath the ground,

where it cannot be examined, it becomes a real element of danger, which may become disastrous without warning, since the decomposition of the slags is produced by organic acids as well as by moisture. Kept free from moisture, mixed wool is one of the best and one of the cheapest materials that can be used for covering steam pipes. To use it successfully it must not become packed, for then it loses its non-conductive power. When it becomes moist it packs, and if this moisture and packing is continuous the slag becomes attacked, liberating its dangerous elements to act on and weaken the pipes.

DISCUSSION.

CHARLES E. EMERY, M. Am. Soc. C. E.—The principal facts stated in this paper were last year presented to the Society of Mechanical Engineers by Prof. Hutton, and fully discussed by the members of that society. Prof. Hutton gave the analysis of Prof. Egleston, who also discussed the paper at that time. The action of organic acids on mineral wool, incidentally added by Prof. Egleston to the original statement, has no bearing on the accident; and the object of duplicating the account of the latter must be that he desires to have it known that he considers mineral wool a particularly dangerous substance, at least under certain conditions. Practical men, knowing their own specialties well, and but little or nothing of chemistry, could, however, inform the Professor that, under the conditions named, the difficulties would occur sooner or later with any kind of fibrous or porous covering. And as this fact is not referred to in the paper, it becomes practically an attack on a special industry.

The New York Steam Company, with which I am connected as engineer, uses mineral wool about its pipes, and the original statements of Prof. Hutton, though well intended, had the effect of an attack on our company in the hands of an opposition steam company then existing, but which soon suspended operations—its representatives advancing the theory that our pipes would soon be destroyed by corrosion. Ere long, however, the theories of our opponents were at a discount; for in attempting to practically carry out a superheated steam theory, attributed by rumor and the patent records to the military branch of the undertaking, the honors of a large explosion at Hell Gate were dissipated in a series of smaller ones in the streets of New York; and as the New York Steam Company felt no solicitude in the use of mineral wool, it was not thought necessary to reply to Prof. Hutton's paper.

In response to the present practical repetition of Prof. Hutton's paper in regard to the explosions at Columbia College, it is sufficient to call attention to the fact that on account of a defective rubber flange packing and lack of means to provide for expansion, several joints of the pipe leaked and admitted moisture into the covering material to such an extent that it sagged down with the additional weight, and in due time the pipes were found to be corroded. I have myself seen precisely the same phenomena with hair felting, sagging and corrosion included; and in

practice such matters are attributed to the leaks, not to the covering material, as corrosion would take place with any covering material such as the various felts, plaster compositions and cements used for covering pipes, when the same are alternately dried and moistened, as in defective heating apparatus. Who does not know how rapidly iron covered with plaster of Paris will be rusted in a damp place?

The principles affecting the corrosion of iron steam pipes and boilers are the same as those for the ordinary corrosion of iron, though generally intensified by the effects of temperature, and in the case of boilers by impurities in the fuel. Wherever a collection of moisture can take place, the water absorbs carbonic acid from the air, and as the solution becomes concentrated by evaporation, the acid attacks the iron, losing a portion of its oxygen in oxidizing or rusting the iron. Other acids will do the same thing, but all the phenomena can be observed in kind, if not in intensity, from the action of the carbonic acid of the atmosphere.

Sulphur in coal promotes corrosion, as seen on the overhead bracing of railroad bridges and in long, damp boiler flues; so also it may hasten corrosion when mineral wool is exposed to moisture, as in the cases under consideration, but, as has been stated, corrosion would take place with any fibrous or porous covering material under the same conditions, and in places where it would not be safe to use mineral wool it would not be safe to use any other unprotected porous material.

The New York Steam Company not only permits no leaks, but never permits its pipes to become cool. The latter is the important safeguard; for, whatever the covering may be, if the same be exposed alternately to heat and moisture the pipes will rust, but so long as they are kept dry this cannot happen. The conduits in which our pipes are laid and surrounded by mineral wool are of brick for the larger pipes and wood for the smaller ones. The brick-work is cemented and covered even down the sides with tarred paper. The wood covering is thoroughly soaked in roofing pitch, and the joints covered with tarred paper, and in moist soil the trenches are drained, all to prevent the absorption of moisture; the same precautions serving, of course, also to keep out the organic acids mentioned by Prof. Egleston.

The brick-work at Columbia College was evidently not protected as above stated, but absorbed moisture from the soil and imparted it to the mineral wool, and the whole structure would remain damp, even when the earth itself was dry, which explains why the unprotected pipe cor-

roded less. If the pipes had been kept either hot or dry continuously, as in our system, there would have been no difficulty. The presence of steam about iron and mineral wool will not produce corrosion. Two short lines of the New York Steam Company were laid with stuffing boxes before the regular fittings of that size were ready. The soft packing soon leaked badly, so that the mineral wool in the conduit, and even the surrounding earth for some distance, was filled with steam, though the same in some places condensed and formed moisture at a little distance from the pipe. These mains have recently been rebuilt, and the pipes found without signs of rust, and in apparently as good condition as new, after being in a steam bath in the presence of mineral wool for considerably over a year.

There is little danger, as claimed by Prof. Egleston, that the wool will become too much packed. Its efficacy as a non-conductor, as pointed out in my paper, to which he refers, is in breaking up the circulation of the air—entrapping it in small cells, so to speak. The smaller the cells the greater the resistance, until the fibres are compressed sufficiently to bridge the cells and offer the least resistance. Mineral wool shoveled over and over, so as to be broken up fine, is still a good non-conductor when dry.

Wet mineral wool does not lose its fibre readily. Bags which in store had become wet were thrown in a vacant lot, and, upon being examined by me, after several years' exposure to storm, frost and sunshine, the fibres were found perfect a little below the surface, and a felted crust had formed at the top, which prevented destruction below. In some of our manholes which were covered improperly, water fell on the mineral wool and felted and matted the upper surface, so as to form a crust which actually became water-tight. In one case a defective Croton service played water on the mineral wool probably for several weeks, when such a crust was formed, and the wool below was found dry.

T. Egleston, M. Am. Soc. C. E.—The circumstances under which this paper was written are all recorded in it. They were fully stated to the Secretary before the paper was offered for presentation to the Society. That two papers on the same subject may not properly be presented to different or even to the same Society, as has been implied, is not a tenable position. It is not only allowable but justifiable, since the subject will usually be presented to a different audience. To forbid it would be to prevent the diffusion of knowledge. It is the constant

practice of learned societies all over the world. There is nothing in this paper which is not distinctly my own. The investigation is wholly mine. I made it for the sake of ascertaining what the facts were, at my own expense, neither for nor against the interest of any persons or company. I never expected it to be considered as an attack on any industry. That such corrosion as would have caused a similar accident would sooner or later be caused by any leak, whether the pipes were covered or not, is true. I did not state this fact in the paper. I saw no reason for doing so, as I supposed that every one knew that iron if exposed long enough to moisture would sooner or later rust through.

It was not my purpose to say what would take place under any or all circumstances, but what did take place under the given conditions in this particular case, and that the accident was caused by the action of the liberated sulphuric acid on the iron of the pipes, as is shown by the analysis of the scale on page 257, and that the action was very much more rapid than it would have been had the steam or water of condensation acted alone. I have no reason to attack any industry, but I did and do propose to call attention to facts of both scientific and practical interest which fall within my professional observation or research. The circumstances under which this accident took place are likely to be repeated. They occur from causes which are perfectly preventable. Slag can be had which does not contain more than a trace of sulphur. The action of organic acids on substances buried in the earth is of the greatest possible importance. Until within a few years their effects have been supposed to be entirely null, and they have been wholly overlooked. That a very weak acid, supposed to be entirely without effect, may, under any conditions, decompose a substance and set free a dangerous element, is of itself alone worthy of the careful attention of every engineer, more especially so when, under the rapid and careless methods of manufacture, the result may be to deface, weaken or even destroy structures. No engineer can afford to ignore the presence of small quantities of acid in sulphur, nor any architect the presence of impurities in plaster or mortar, and there is just as much reason for calling attention to the action of alkalies and of very weak organic acids. I do not know, and did not pretend to know, the practice of the New York Steam Heating Company. I know nothing about it, except as I have seen it while passing through the streets of New York, where their pipes are laid. But I do know what

happened at Columbia College, and have described its cause. The statements I have made in both papers are the results of my own observations and researches. Whether they are or are not of value cannot be determined by any assertion. This Society cannot lend itself, and no member of it can afford, to depreciate or discourage, however slightly, original research. Many years' experience with, and investigation of, blast furnace slag have taught me to appreciate its value in very many ways, and as a safe article can be had just as well as a more or less dangerous one, it is the right of every member of the Society to know under what conditions it may be perfectly safe, and under what circumstances it may present elements of danger. If it is true that all blast furnace slags, when reduced to the condition of fibre, are decomposed sufficiently by simple atmospheric influences to form a more or less solid substance, as it is asserted, the infiltration of water through the "felted" substance must dissolve out both alkalies and sulphur, to be ready to attack whatever comes in their way.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
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TRANSACTIONS.

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CCLX.

(Vol. XII.—July, 1883.)

GEODETIC FIELD WORK.

By GEO. Y. WISNER, M. Am. Soc. C. E.

READ APRIL 4TH, 1883.

The system of land surveys of this country is based on such inaccurate methods that in nearly all the older States, where the land has greatly increased in value, the original surveys are no longer a suitable basis for subdivision of property.

Since old boundary marks cannot be disturbed, resurveys merely determine their true positions, without correcting the irregularities of boundary lines; and consequently it would be much more satisfactory as well as economical if the original surveys were based on permanent marks established by an accurate system of triangulation.

The resurvey of States, and the development of geodetic surveys by the General Government, are having a tendency to make a knowledge of geodesy of more importance to the engineers of this country than heretofore, and will probably eventually make it as prominent a specialty with the profession as at present with the engineers of Europe.

The object of the following paper is to present, in as concise a manner as possible, practical methods of making geodetic observations, and to point out some of the sources of error which are liable to escape the notice of the inexperienced observer.

The different text-books on this subject are mostly of a theoretical character, and although they treat of a variety of methods of observation, give but little of the practical information needed for doing successful work.

An erroneous result, due to the use of an incorrect formula, may at any time be revised and corrected, but an error of observation once a matter of record often renders work worse than useless.

The mere fact that the individual results from a series of observations agree with each other is by no means proof that the work has been accurately done, for this agreement may be due to the accidental errors of observation being small, while those errors which are constant as to sign have no effect on the sign of the residuals, and are often undetected until the final discussion of results produces discrepancies which may render the work of but little value.

The labor of computing the most probable corrections to results for geodetic field observations is often a more difficult task than to make the observations, and there are unfortunately plenty of examples where the results thus corrected are not nearly so reliable as a mean value would have been had the observations been so made as to eliminate constant errors.

Before commencing any important series of observations, it is essential that the observer should accurately determine the instrumental constants of the instruments to be used, as injury to reticule threads and level tubes are of frequent occurrence, and unless the values of thread intervals and level divisions are known, an accident may render a large amount of work worthless.

The observer should also be prepared to replace reticule threads in case of breakage, as the operation is exceedingly simple, and to be obliged to send an instrument to the maker's, as is often done, is expensive, and necessitates a loss of time.

A cocoon of fine spider thread, a pair of spring dividers, and a bottle of shellac dissolved in alcohol, is the only outfit needed.

A thread of the cocoon, fastened to the points of the dividers, may be stretched one-third of its length if held in steam and gradually subjected to tension, and if then fastened in position on a diaphragm with shellac, there will be little danger of change of position taking place, even though used in moist atmospheres.

Astronomical Longitude.

The difference in longitude of two places is the interval of sidereal time elapsing between the passage of the meridians of the two places by a fixed star, or difference in local times of the two places.

The method of determining this interval usually consists in observing for local time at the respective stations, and then, by means of signals that may be observed simultaneously at both stations, determining the difference of these times.

Since the telegraph furnishes a ready method of making such comparisons, the determination of local time comprises the principal work in making observations for difference of longitude. As a knowledge of local time is also necessary in both latitude and azimuth observations, it becomes one of the most essential features of geodetic work.

The best form of instrument at present for general use in determining local time, is probably the ordinary astronomical field transit, consisting of a telescope of about 30-inch focal length with a horizontal axis supported in the wyes of an iron frame. Two small circles, with verniers reading to minutes, are attached to the opposite sides of the telescope, near the eye-piece end, and enable the observer to set telescope at any required altitude. One of the wyes may be moved in azimuth by a slow motion screw, for the purpose of making the final adjustment for azimuth after the instrument has been placed approximately in the meridian.

The axis of the telescope is hollow, allowing the light from a lamp supported on a bracket attached to one of the wyes, or at side of observatory, to pass into the telescope to a reflector at the intersection with axis, from which it is reflected to the threads of reticule. The practice, however, of supporting the illuminating lamp on the instrument is bad, and should be avoided if possible, as the heat produces unequal expansions of different parts of the transit and striding level, and consequently may cause the level to indicate erroneous corrections.

The transit, as usually constructed, can be reversed only by lifting the telescope by hand from its wyes and shifting the pivots into the opposite supports. This cannot be done without more or less jar, and is liable to disturb both the azimuth and collimation adjustments. A secondary set of wyes, supported by a vertical axis in lower part of iron frame of transit, and so arranged that by means of a screw the telescope

may be lifted from its bearings and turned 180° in azimuth, adds greatly to the ease and precision with which the instrument may be manipulated.

The German form of field transit is somewhat similar to an ordinary altitude and azimuth instrument, except that at the intersection of the telescope and its axis is placed a right-angled prism to reflect the rays from the object glass to one end of the axis, to which the eye-piece of telescope is attached.

With this form of instrument the observer does not have to change position in observing stars of different altitude, which is a great convenience, but it has defects which are serious objections to its use.

The weight of telescope and its counterpoise produces considerable flexure in the rotation axis, which will be in the plane of the telescope and axis when the object sighted at is in the zenith, and at right angles to this plane when object is in horizon.

The collimation of instrument will therefore vary with the altitude of the object observed.

A convenient arrangement for threads of reticule for time observations is in three sets of five threads each, so arranged that the intervals in the sets will require about two and one-half seconds for the passage of an equatorial star, and the interval between sets about double that between threads—the middle thread of the middle set being in the collimation axis when the instrument is in adjustment.

The interval of each thread from the collimated thread may be determined by observing the times of transit of a circumpolar star across the reticule.

If δ = declination of star, i = equatorial interval of any wire, and I = the observed interval from transits of star ; Sin. i = sin. I cos. δ (1).

For stars with polar distances exceeding 10°, $i = I \cos. \delta$ is sufficiently accurate.

If n = number of threads observed over for any star, the correction to the middle thread for the mean of the observations will be

$$\frac{\Sigma i}{n} \sec. \delta. \qquad (2.)$$

Let φ = latitude of place, δ = declination of star,
a = azimuth constant, b = level constant,
c = collimation constant, and $\triangle t$ = clock correction, due to instrumental errors, it may be easily shown that

$\triangle t = a \sin.(\varphi - \delta) \sec.\delta + b \cos.(\varphi - \delta) \sec.\delta + c \sec.\delta$ * (3).

If $180° - \delta$ be substituted for δ in formula (3), the resulting equation will give corrections for stars observed at lower culmination.

From the general formula (3) it will be seen that the correction due to the inclination of the axis varies with the co-sine of the zenith distance of star, and as it is impossible to maintain the axis exactly in a horizontal position, it becomes necessary to determine the amount of this inclination at frequent intervals during the observations.

Let e, e^1 and w, w^1 be the respective readings of the east and west end of bubble for direct and reversed positions of the striding level, and we have $b = \frac{(w + w^1) - (e + e^1)}{4} d$, where d is the value of one division of level scale, and may be determined in various ways.

The most convenient method of determining the value of level division is by means of a level trier, but if such an instrument is not at the disposal of the observer, nearly as good results may be obtained by comparisons with the micrometer screw of a zenith telescope. If the level be placed parallel to the optical axis of telescope, and the latter pointed to a target near the horizon, by moving the telescope in altitude so that the bubble runs from one end of scale to the other, and noting the arc passed over by reading on target with micrometer for each position, the value of division may be obtained in terms of the micrometer revolution.

Another method consists in measuring a line of 500 feet to 1 000 feet length from the observing post and setting up a leveling rod at other end of line, so that by means of the target the vertical distances passed over by horizontal thread from slight motions of telescope may be accurately read on the rod.

By placing the level on telescope of transit parallel to its optical axis and moving it in altitude till bubble reads near one end of scale, and noting the target reading on level rod corresponding to this position, and then moving telescope till bubble reads at other end of scale, the arc passed over may be computed from the target readings on the level rod. If n = number of divisions passed over by the bubble, a = length of base and b = the vertical distance through which level target was moved, $nd = \text{tan.}^{-1}\frac{h}{a}$ (4). This should be repeated a number of times

* Chauvenet's Astronomy, Vol. II., p. 145.

to eliminate errors of observation, and also on different parts of the scale, to ascertain if all of the divisions are of equal value.

The pivots of the axis often have unequal diameters, in which case the result obtained with striding level does not give the true inclination of rotation axis, and must be corrected for the error due to this inequality.

Let p = pivot correction, $2\,\Theta$ = the angle of the wyes of level, $2\,\Theta^1$ = the angle of the transit wyes, and b and b^1 the inclination of the axis as given by striding level for direct and reversed positions of telescope, then $p = \frac{b^1 - b}{2}\left(\frac{sin.\ \Theta^1}{sin.\ \Theta + sin.\ \Theta^1}\right)$ (5.)

If the angles of the wyes for transit and level are approximately the same equation, (5) becomes $p = \frac{b^1 - b}{4}$.

This error may be nearly eliminated from the mean result by observing about an equal number of stars for each position of the telescope, but whenever of sufficient magnitude to affect the results the correction should always be applied.

In order that the central thread of the reticule may revolve in the same plane for direct and reversed positions of the telescope, the reticule must be so adjusted that the vertical plane passing through the central thread and optical axis of telescope will be at right angles to the rotation axis. Since this adjustment can seldom be made absolutely correct, and as it is also liable to change, the error should be made as small as possible by carefully collimating the central thread by reversing the telescope on a distant target, and then determine the remaining error and apply it as a correction to results.

If a circumpolar star be observed over the first set of threads of reticule with telescope direct, and the telescope then be reversed, the star may again be observed over the same threads in reverse order.

These results, when reduced to middle thread by formula (1), and corrected for level error, should differ by twice the collimation error multiplied by secant of declination of the star, or, $C = \frac{t - t'}{2\ sec.\ \delta}$ (6), where t and t' are the observed chronometer errors, before and after reversal. The mean result from t and t' is evidently free from collimation error, and if combined with the result from an observation on an equatorial star will give the azimuth constant.

Suppose the deviation of the instrument is to the east of south, and that t and t' are the chronometer corrections obtained from observations on a north and a south star, and $\triangle t$ and $\triangle t'$ are the corrections to those results due to the azimuth error.

Then, since $\triangle t = - a \sin. z \sec. \delta$ and $\triangle t' = + a \sin. z' \sec. \delta'$

$$a = \frac{\triangle t' - \triangle t}{\sin. z \sec. \delta + \sin. z' \sec. \delta'} = \frac{t' - t}{\sin. z \sec. \delta + \sin. z' \sec. \delta'} \quad (7).$$

The two terms of the denominator are usually called azimuth factors, and may be tabulated for different declinations and zenith distance, so that with declination and zenith distance of a star for arguments, the value of the factor may be taken directly from table.

In computing the azimuth constant, it is necessary that the observations be first corrected for the level and collimation errors, and also for rate of chronometer, if much time elapses between the observations used. In case both the stars are on same side of the zenith and at upper culmination, the denominator of equation (7) becomes the difference of the factors; and for a star at lower culmination, the sign of its factor will be the same as for a star south of zenith.

With a good instrument and proper care in manipulation, the constants for azimuth and collimation will remain substantially the same throughout a night's observations; but for precise work, pains should be taken to eliminate their effect as nearly as possible from the resulting chronometer error.

The value of b obtained from the level readings, together with the mean value of a and c deduced from the observations, when substituted in formula (3), give the correction for each star due to instrumental errors.

The post on which the instrument is placed should be set at least 5 feet in the ground, to insure stability, and have a small observatory built over it, to protect the instrument from the wind.

When the reticule is properly adjusted the horizontal thread should be parallel to the horizon, and if the instrument is in the plane of the meridian the stars should cross the reticule parallel to this thread. Hence, if the transit be placed approximately in the meridian by estimating from the hour angle of Polaris the amount of its deviation, and an observation be then made on a time star, the direction in which the star crosses the horizontal thread will indicate which way the telescope deviates from the meridian.

If the star crosses the horizontal threads upwards, the telescope will be pointing to the eastward of the meridian, and if moved in azimuth by slow-motion screw of movable wye until the star follows the threads, the instrument will be approximately in the meridian. If the star moves downwards in crossing the threads the telescope will have to be moved in azimuth to the eastward, and then tested by another star.

It is not necessary to set the instrument for any particular star, for if the telescope be moved slowly in altitude a star may be easily found suitable for testing this adjustment.

When the transit is so near in the meridian that a time star will follow the horizontal thread in crossing the field of view, if the time of transit of a star within a few degrees of the zenith be observed, the result will be affected but slightly by the azimuth error. An observation of this kind will give the approximate error of the chronometer; and if the telecope be then set for a circumpolar star, and moved in azimuth until the collimated thread of reticule bisects the star at the time for its culmination, the transit will be sufficiently near in the meridian for making observations.

In case a closer approximation is desirable, the chronometer error may be redetermined by observing another time star, with which the azimuth of transit may be adjusted by setting thread on another slow star, as before.

Where several nights' observations are to be made, a meridian mark may be established from the first night's observations, so that the transit may be placed in the meridian on other nights by simply sighting to mark.

The difference between the right ascension of a star and its observed sidereal time of transit is the amount that the chronometer is either fast or slow.

The positions of the stars being given in the ephemeris by their right ascensions and declinations, it will be necessary to know the approximate lattiude of place before zenith distances can be computed for setting telescope; and if a mean-time chronometer be used, the right ascensions of the stars observed should be converted into mean time.

Since the sidereal time of mean noon given in ephemeris is for the meridian of Greenwich, to obtain the sidereal time of mean noon for place of observation, $\overset{sec.}{9.8568}$ must be added to the ephemeris time for each hour of longitude west of Greenwich.

If from the right ascension of a star, as given in ephemeris, the sidereal time of mean noon be subtracted, the difference will be the sidereal interval of time from mean noon to time of culmination of star at place of observation.

This interval, converted into mean time by use of tables, or by subtracting $\overset{sec.}{9.830}$ for each hour past noon, will be the mean time of culmination of star; and if subtracted from the observed mean time of transit, the difference will be the chronometer error.

In observing the times at which a star crosses the threads of the reticule, the observer may either note the times of transit by listening to the beats of the chronometer and estimating between what seconds the bisection takes place, or record the observations automatically by means of a chronograph. By the former, or eye and ear method, the star is usually observed over the first and last threads of the outside sets and middle thread of middle set, except for slow stars, on which the telescope is reversed, in which case the star is observed over the threads of an outside set in opposite directions for the two positions of the telescope.

In recording the times of observations by means of a chronograph, a break circuit clock or chronometer is required, consisting of an arrangement of the works so that the circuit from a small battery, passing through the chronometer, is broken an instant at each second, except the zero second of each minute.

The wire from one pole of the battery passes through the break circuit of chronometer and is connected with the coil of a small relay, to the armature of which the chronograph pen is attached, and then passes through break circuit key at observing post back to battery.

The chronogragh consists of a cylinder about 6 inches in diameter, with clock-work attachment, by which it may be turned at the rate of one revolution per minute.

The pen held by armature of relay rests on a sheet of paper fastened around the chronograph cylinder, and for each revolution of the cylinder the relay and pen are drawn sidewise about $\frac{1}{10}$th of an inch. Each beat of the chronometer breaks the circuit for an instant, and causes the pen carried by armature to make a slight break in line marked on the chronograph cylinder.

Since each of these breaks corresponds to some second of time, if the

observer breaks the circuit with the key at observing post at the instant the star crosses any thread of the reticule, the break will be recorded on chronograph, and the distance of this break from the last preceding break made by the chronometer will give the exact fraction of a second at which the observation was made.

The same circuit used for recording the observations on the chronograph may be connected with the wires of a telegraph line, so that the breaks of the chronometer may be recorded on a chronograph at some other station for the purpose of comparing the local time and determining the difference of longitude.

If an ordinary Morse relay be placed in the circuits, so that the local line passes through the coils and the main line through the armature of the relay, the second breaks from the chronometer will be repeated over the main line, and if at the other station the main line be switched to pass through the coils, and local line through the armature of relay, the breaks will be recorded on the chronograph with those from the chronometer at that station.

The results from these records will be in error by the amount of time required for the electric current to pass over the lines and act on the armature of the relays, but since the signals sent from the easterly station give too small a difference of time, and those from the westerly station too large a difference, the mean result from signals sent in both directions will be free from wave and armature time.

In mountainous regions, and where the observers are unable to make comparisons by means of telegraph, the differences of local time may be obtained by making simultaneous observations of powder flashes made at some place visible from the respective stations.

The time at which any flash occurs may be easily observed to the nearest tenth of a second, and consequently a mean of twenty such comparisons will be sufficiently accurate for each night's observations.

These comparisons, whether electric or otherwise, should be made each night, so that about one-half of the time observations will precede the signals.

Since the difference in longitude of two places is the sidereal interval of time of a star in passing from meridian of one place to that of the other, it follows that if the same star be observed at each station all errors in right ascension will be eliminated from the result for difference of longitude.

If a sufficient number of well-determined slow and time stars be observed to establish the instrumental constants, any time star, if observed at both stations, will give a good result for difference of longitude.

This allows a much greater number of observations to be made in a given time than would be possible if only standard stars were observed.

Since no two observers will estimate the time of bisection of a star at exactly the same instant, the difference in longitude obtained from the local time determined by two observers will need a correction due to the difference of their personal errors. This correction may be determined for any set of observers by making independent determinations of local time at stations near each other, the difference of which, when corrected to the same meridian, will be the personal error of the observers affecting the difference of longitude.

This error, however, is liable to change with age and practice, and should not be assumed to be constant for any great length of time.

If, after completing one-half the observations required for a longitude determination, the observers change stations and complete the work, the mean result will be free from this source of error.

Diurnal aberration causes each star to appear to the eastward of its true position by an amount $= \frac{sec.}{0.021} \cos. \varphi \sec. \delta$. This correction is additive to observed time of transit of upper culmination stars, and subtractive for stars at lower culmination.

The instrumental constants deduced from observations of high and low stars substituted in formula (3) will give the individual results with sufficient accuracy for ordinary work, but where greater precision is required, then the azimuth and collimation constants and chronometer error may be deduced from the observations by the "method of least squares."

Let $\triangle t$ = assumed approximate chronometer error at time of signals,

$\triangle t'$ = difference between assumed and true chronometer error,

R = interval of time from mean of signals to time of any observation,

ρ = hourly rate of chronometer,

v = residual obtained by subtracting the clock error for any observation from the assumed chronometer error $\triangle t$.

Each star observation will then give an observation equation of the

form $Aa + Cc + R\rho + \triangle t' = v$, and these may be solved by the usual method of least squares.

In case there is any reason to suppose that the azimuth or collimation of the instrument was disturbed during the observations, the equations must be solved in sets for values of the constants before and after the disturbance occurred.

The probable error of a good longitude determination is about $\overset{sec.}{0.03}$, or an amount corresponding to about 45 feet at the equator. Owing, however, to the unequal density of the earth's crust and the irregular shape of the geode, the direction of the action of gravity often makes a slight angle with the corresponding meridian plane of the most probable spheroid, and consequently the transit when accurately leveled will revolve in a plane slightly inclined from the normal to spheroid.

The effect of this deviation is to produce discrepancies in the resulting difference of longitude that may amount to $\overset{sec.}{1.0}$ or more, as shown by a comparison of the observed difference with difference computed geodetically.

The simplicity with which time may be determined from observations of star transits, and the fact that a few such observations with an ordinary theodolite will determine time to the nearest second, enables any engineer to furnish the correct time for the railroad or city he may represent.

Where local time only is required, the engineer, by establishing a meridian mark, may place his instrument accurately in meridian by setting collimated thread on mark, and the observation of a few time stars for both positions of telescope will then give a mean result sufficiently accurate for rating clocks.

Astronomical Latitude.

The method of determining latitude by means of zenith telescope is probably the most accurate now in use for geodetic field work.

Let z and δ be the zenith distance and declination of a star culminating south of the zenith, and z' and δ' for a star culminating about the same distance north of the zenith, then

$$z = \varphi - \delta \quad (8) \text{ and } z' = -\varphi + \delta' \qquad (9.)$$

$$\therefore \quad \varphi = \tfrac{1}{2}(\delta + \delta') + \tfrac{1}{2}(z - z') \qquad (10.)$$

Therefore, if the difference of zenith distances of two such stars be measured and one-half be applied as a correction to the mean of the declinations, the resulting latitude will be affected by only the errors due to diurnal aberration and the difference in refraction for the two stars; and since the stars have nearly the same altitude, the latter will always be small.

The instrument consists of a telescope about 45 inches focal length, mounted at the end of a horizontal axis which is supported in bearings at top of a vertical axis, so that the telescope may be set at any altitude by means of a setting circle and level, and then turned in azimuth on vertical axis so as to point either north or south of the zenith, as required.

The horizontal axis is hollow, allowing the light from a lamp at end of axis to pass to a reflector at intersection with telescope, by which it is reflected to cross threads of micrometer.

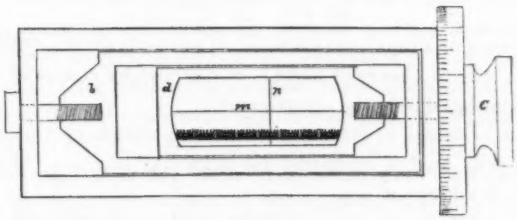

Fig. 1.

The micrometer consists of a frame, *b*, in which a slide, *d*, carrying a fine spider thread, may be moved in a plane at right angles to the optical axis of telescope by means of the micrometer screw, *c*.

The centre of the micrometer slide is cut away so that the micrometer thread, *n*, is projected on a scale at lower edge of opening, the divisions of which correspond to the distance passed over by the thread for each revolution of the screw.

The spider thread, *m*, is attached to the frame, *b*, and when adjusted, should be in collimation axis of the telescope. Every fifth notch of the scale is made deeper than the others, to enable the observer to read the number of the revolution quickly for any position of micrometer thread. The head of the screw is graduated into one hundred divisions, and numbered so that the numbers will increase past the index

as the micrometer thread moves in a positive direction over the scale, or away from end of scale, corresponding to the zero revolution.

The value of the space passed over by the micrometer thread for each revolution of the screw may be readily obtained by observing a circumpolar star when near elongation across the thread for positions corresponding to each revolution of the head. If the instrument be carefully leveled in the plane of the star when near elongation, and the micrometer thread set at that end of scale from which the star will cross the field, the time of transit may be observed for each revolution by moving the thread forward after each bisection.

Readings of the level attached to setting circle of the telescope will indicate the amount of change that takes place in the inclination of the axis during the series.

Let t be the difference of time of any observation from time of elongation of star, and z the value of the space in arc, then

$$z = \frac{cos.\ \delta\ sin.\ t}{sin.\ 1''} \qquad (11.)$$

The values of z thus obtained must be corrected for the difference in refraction due to change of altitude of star, and for change of inclination of telescope during the observations.

If $\triangle \gamma$ = change of refraction for 1'.0 at altitude of star, each observation will need a correction for refraction $= z \triangle \gamma$ where z is expressed in minutes of arc.

If $N_\circ$ and $S_\circ$ be the readings of the north and south end of level scale for the observation at zero revolution, the correction for change of inclination for the n revolution will be $\left(\frac{N_\circ - S_\circ}{2} - \frac{Nn - Sn}{2}\right) d$, where d is the value of one division of level scale, and may be determined from a comparison with micrometer screw. The values of z when subtracted successively from each other give the corresponding values of the micrometer revolutions, and should be deduced from the mean of at least four transits of circumpolar stars. In some micrometers the value of revolution varies for different parts of the screw, in which case care must be taken to use the value of revolution corresponding to the part of scale on which the difference of zenith distance is read.

The post on which the instrument is supported should be set firmly in the ground, with an observatory built over it to protect instrument

from the wind. By using the delicate level on setting circle for making adjustments, the instrument may be so accurately leveled that the bubble will move through but a few divisions when the instrument is turned in azimuth, which is essential where precision is required.

The instrument having been leveled by means of setting level, the horizontal axis may be adjusted by means of striding level and adjusting screw near outer end of axis. This adjustment is not always stable, and should be tested at intervals during the work. Since the telescope is at one side of vertical axis, it is necessary in collimating vertical thread to make reversals on a distant target, or else have two targets separated by twice the distance of telescope from the vertical axis.

The micrometer box is fastened to the telescope tube with a clamp and adjusting screws, with which the micrometer thread may be made parallel to the horizon when the instrument is level.

The instrument may be placed in plane of meridian by the same process described for the field transit, and when so adjusted, if the two stops attached to horizontal circle be firmly clamped against the tangent arm, the telescope may be quickly brought into the meridian either north or south by turning instrument in azimuth until tangent arm rests against one of the stops. Care, however, must be taken that no dust accumulates between the stops and tangent arm during the observations.

If the vernier of setting circle be set at the mean zenith distance of the pair of stars to be observed, and the telescope moved in altitude till the bubble of setting level comes to the centre of scale, and then firmly clamped, the stars will cross the field of view about equidistant from centre of micrometer scale. As the first star of pair crosses the vertical thread it should be bisected by the movable micrometer thread and the time noted. The position of thread on micrometer scale will give the number of revolutions, and the division of head under the index the hundredths of revolution for the observation. The level should then be carefully read and the telescope turned 180° in azimuth, and the other star of pair observed in a similar manner.

If d is the value of one division of the level scale, and n n' and s s' are the respective readings of the north and south ends of the bubble for the two positions of the telescope, each latitude result will need a correction for inclination of vertical axis $= \frac{(n + n') - (s + s')}{4} d$ (12).

If the sum of the north readings is the larger, the vertical axis points to the south of the zenith, and the correction is additive to the latitude result. If the south readings are the larger, the correction must be subtracted. When the instrument is in the meridian, and a star is observed off the collimated thread, the latitude result will need a correction $= + \frac{1}{2} (15 \triangle t)^2 \sin. 2\delta \sin. 1''$ for each star, where $\triangle t$ is the hour angle of the star at the time of bisection.

If the telescope is not in meridian, and the bisection is made when star is on collimated wire, the correction

$$= - \frac{(15 \triangle t)^2 \cos. \varphi \cos. \delta \sin. 1''}{4 \sin. (\varphi - \delta)}$$

Unless $\triangle t$ exceed five seconds these corrections will not materially affect the result.

When the difference of zenith distances of a pair of stars amounts to several minutes, a correction for refraction must be applied to each result, equal to one-half the difference of zenith distances, multiplied by change of refraction for one minute at altitude of stars.

The United States Engineer and Coast Survey officers have published catalogues of stars suitable for forming lists of pairs for most places in the United States.

In making up a list of pairs much labor will be saved by preparing a table of declinations of north and south stars corresponding to different zenith distances.

By entering table with the declination of any star, as given by catalogue, the number in corresponding column will be the approximate declination required for a star to complete the pair.

Stars should be selected, if possible, that do not differ more than 10 minutes in right ascension and 30′ in declination.

Thirty to forty pairs of stars may be easily observed in one night, and if obtained on three different nights, should give a mean latitude result having a probable error of less than 0″.10.

Since the probable error of the mean declination of a pair of stars is greater than the probable error of a single determination of the difference of the zenith distances, but little precision will be gained by observing the same pairs on more than three different nights.

When a mean time chronometer is used for observing the times of transit, the approximate mean time of culmination should be computed for the stars for the different nights. The deviation of the direction of

plumb line from the normal to the most probable spheroid produces discrepancies between the computed and observed latitude in geodetic work which in extreme cases may amount to 15″.0 or more.

Astronomical Azimuth.

To make a complete determination of the azimuth of a line requires observations for local time and latitude, and the measurement of angles under unfavorable circumstances. The method usually adopted is to measure the angle between a signal at the opposite end of line of which the azimuth is required, and a circumpolar star when near its elongation, and from the hour angle of the star at time of observation compute the azimuth. Let the telescope be placed in the plane of the star s when

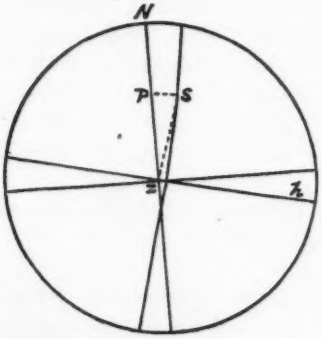

Fig. 2.

near its elongation, and let z be the zenith of the place, P the pole, and h the direction of the rotation axis; then $z\,h = 90° - b'$, and $3\,h = 90 - c$, where b' is the inclination of rotation axis and c the collimation of cross thread, assumed to be to the left of the optical axis of the telescope. Letting a' = the azimuth of the star, and $a_{,}$ = the azimuth of rotation axis, the triangle $s\,z\,h$ gives $-\sin.\ c = \sin.\ b'\cos.\ z' + \cos.\ b'\sin.\ z'\cos.(a_{,} - a')$. Since $a_{,} - a'$ differs but little from 90°, and the inclination of axis and collimation seldom exceeds a few seconds, the above equation may be written:

$$a' = a_{,} + \frac{c}{\sin.\ z'} + \frac{b'}{\tan.\ z'} - 90°$$

If a reading be made to another object having a zenith distance z'', and azimuth a'', the equation becomes :

$$a'' = a_{,,} + \frac{c}{sin.\ z''} + \frac{b''}{tan.\ z''} - 90^{\circ}$$

in which $a_{,,}$ and b'' are the azimuth and inclination of rotation axis. The difference of these equations give :

$$a'' - a' = a_{,,} - a_{,} + c\left(\frac{1}{sin.\ z''} - \frac{1}{sin.\ z'}\right) + \frac{b''}{tan.\ z''} - \frac{b'}{tan.\ z'} \quad (13).$$

Since small errors of inclination do not produce measurable errors in the readings to objects near the horizon. the angle $a_{,,} - a_{,}$ will be the difference of readings on the horizontal circle corresponding to the two positions of the axis ; and if one of the objects pointed at be near the horizon, equation (13) becomes :

$$a'' - a' = a_{,,} - a_{,} + c\left(\frac{1}{sin.\ z''} - 1\right) + \frac{b''}{tan.\ z''} \quad (14).$$

in which $a'' - a'$ is the angle between the objects, and $a_{,,} - a_{,}$ the difference of the readings on the horizontal circle.

Let t = the hour angle of star at time of observation, and A = its azimuth ; then from the triangle $Z\ P\ S$,

$$-\ tan.\ A = \frac{sin.\ t}{cos.\ \varphi\ tan.\ \delta - sin.\ \varphi\ cos.\ t} \quad (15).$$

At elongation the triangle is right-angled at s and $cos.\ t. = \frac{tan.\ \varphi}{tan.\ \delta}$ (16 $sin.\ A = \frac{cos.\ \delta}{cos.\ \varphi}$ (17).

Putting p = the angle at star between the pole and zenith and differentiating equation (15) with reference to φ, t, and δ, and we have $dA\varphi = -\frac{sin.\ A\ d\ \varphi}{tan.\ z.}$; $dA_{t} = \frac{cos.\ \delta\ cos.\ p}{sin.\ z} dt$; $dA_{\delta} = \frac{sin.\ p}{sin.\ z}\ d\ \delta$, from which it may be seen that an error in time will have a maximum effect on the azimuth at culmination, and a minimum effect at elongation of star, while errors due to latitude and declinations produce a maximum effect at elongation and a minimum effect at culmination.

Stars observed at lower culmination have the advantage of being at a less altitude than when observed at elongation; and since at culmination of Polaris an error of one second of time only produces an error of 0".3 in the computed azimuth, it is evident that if the chronometer error be well determined, observations on close circumpolar stars at lower culmination will be subject to errors but little larger than those that may occur in observations made at time of elongation.

The declinations of the four standard circumpolar stars given in the "American Ephemeris" are by no means accurate, and, consequently, for precise work, observations should be made on stars at both elongations to eliminate the effect of declination errors from the azimuth result.

In the discussion of the results for azimuth in the geodetic work of the Lake Survey discrepancies were found that could be accounted for only by supposing the declinations of these stars in error. The Germans have since published corrections for these declinations which materially reduce the size of the discrepancies; but the results still indicate that other corrections are needed.

When observations are made to circumpolar stars near elongation the latitude of place should be determined to within less than 05".0, or otherwise the error in computed azimuth, due to this cause, may be larger than those due to errors of observation.

If pairs of stars be selected in a similar manner as for zenith telescope, and the differences of their zenith distances measured by means of the vertical circle and setting level of the theodolite, the resulting latitude will be practically free from errors due to refraction and errors of graduation. A mean of ten such results from observations with a theodolite having a vertical circle reaching to 05".0 will usually give the latitude within the limits required for the reduction of azimuth observations.

In case the theodolite is supplied with a micrometer eye-piece it may be used in precisely the same manner as a zenith telescope, and an accurate determination of latitude made.

The great precision of the non-repeating theodolite, reading with micrometers, and the ease and rapidity with which it may be manipulated, makes it especially adapted for azimuth observations. The essential difference of this form of instrument from an ordinary theodolite is that, instead of an inner circle carrying verniers, two or more equidistant arms project from the support of the wyes and carry microscopes with screw micrometers, so adjusted that when the graduations of the circle are brought into the focus of a microscope one revolution of the micrometer screw will move the micrometer thread over an aliquot part of the space between two graduations.

The form of micrometer is the same as that used on the zenith telescope, except that the head of micrometer screw is graduated into sixty parts to correspond to seconds of arc, and numbered so that the readings will increase as the thread moves backward over the graduation of the circle.

The opinion seems to be prevalent that the accuracy of geodetic work depends on the use of very large theodolites. This, however, is erroneous, for, if the instrument is of sufficient weight to be stable, but little precision can be gained by increasing the size of the circle beyond the limit where the probable error of the mean micrometer reading is less than the probable error of making a bisection with the telescope.

With ordinary atmospheric disturbance, the probable error of a single bisection of a distinct signal with a good telescope is about 0".4, while the probable error of mean readings of micrometers on a good ten-inch circle seldom exceeds 0".2.

The comparison of observations for azimuth, and the measurement of angles made with different sized circles, show that theodolites with 10-inch circles give fully as precise results as those having circles of three times that diameter. For secondary work, where less precision is required, non-repeating theodolites, with circles from 6 to 8 inches in diameter, will be found much better adapted to the work than instruments of larger size.

With a micrometer theodolite a reading consists in measuring with the micrometer the amount that the zero of the scale (centre notch) is beyond the last graduation mark passed by a microscope as the instrument is turned in a positive direction. If a micrometer thread be made to coincide with the graduation mark, the notches of scale between thread and zero notch will indicate the minutes, and the reading of micrometer head the seconds, that must be added to the reading of the graduation to give position of the zero of micrometer scale.

The best arrangement for micrometer threads is two parallel threads at such distance apart that a graduation may be seen distinctly between them.

In order that the micrometer shall indicate the exact second for any reading, it is necessary that it be so adjusted that the thread will move over an aliquot part of a space on the circle for each revolution of the screw. To make this adjustment, let the instrument be firmly clamped and consecutive readings made on two graduation marks under the microscope. If the number of revolutions made in moving the thread backward over the space is too small, the microscope tube is too short, and should be adjusted by loosening the set-screw of objective and screwing it out a few turns. By making another set of readings on the graduation marks a simple proportion between the results from the two

sets, and the number of turns given the objective will give the amount of adjustment still needed.

This adjustment requires that the actual value of the space between the graduation marks be known, and since the graduation is subject to both accidental and periodic errors, this value should be determined from comparison with the mean of a number of spaces equally distributed around the circle.

By comparing the test space with the zero space of each 10 degrees around the circle, the mean value obtained from the difference may be assumed to be practically free from the effect of accidental and periodic errors of graduation. The test space should be read over for each comparison, otherwise any change in value of revolution would produce errors in the differences.

Let a, a', etc., be the difference between test space and each even ten degree space around the circle, and $\triangle s$ be the correction to the test space, then $\triangle s = \frac{\Sigma a}{36}$.

Changes of temperature produce slight variations in the distance of objective from the circle, causing corresponding changes in the value of the revolution, which should, therefore, be determined for each series of observations.

Let s = mean value of a space on the circle,

$\triangle s$ = correction of the test space,

s' = value of test space given by micrometer,

γ = correction for run of micrometer for 1′.0.

Then $\gamma = \frac{(s \pm \triangle s) - s'}{s}$ in which the upper sign is to be used when the test space is larger than the mean space.

An error frequently made is to give this correction an erroneous sign.

Since the numbers on the micrometer head increase as the thread moves backward over the circle, the reading made on the second graduation mark of test space should be subtracted from that made on the first. The value of s thus obtained, when substituted in the above formula, will give the value of γ with its proper sign, which, multiplied by the number of revolutions of any reading, gives the correction for run.

The errors to which azimuth observations are liable may be divided into two classes—accidental and instrumental.

The former are usually due to atmospheric disturbance and errors of reading, and can only be eliminated by taking such a number of observations that the mean will probably be free from their effect. The latter arise from errors of graduation and from imperfect adjustments of the instrument, and should be eliminated by arranging the observation so that the algebraic sum of the corrections for the final result will be zero.

The periodic errors of graduation may be expressed by the following formula: $-Ex = a_1 \sin.(x_1 + b) + a_2 \sin.(x_2 + b_2) +$, etc.,* in which Ex is the periodic error corresponding to the reading x, and a_1 a_2 b_1 etc. are constants depending on the instrument.

If an instrument is read by means of n equidistant microscopes it may be shown that the mean of the n readings will be effected by only that portion of the periodic error due to the terms of the equation involving nx and multiples of nx, and that if the circle is shifted after each set an amount equal to the angular distance between microscopes divided by the number of sets it is proposed to take, the mean result will be practically free from the effect of periodic error of graduation. If it is proposed to observe six sets, the circle should be shifted approximately 30° after each set for a two microscopic instrument and 20° after each set for a three microscopic instrument.

It is not desirable, however, to shift the circle exactly the same amount for each setting, as a difference of a few minutes will have no practical effect on the elimination, and slight ranges in the settings will have a tendency to give opposite signs to the correction for run of micrometers for the different sets.

The greatest source of error in azimuth observations is probably due to lack of precision in determining the inclination of the rotation axis.

The correction to the inclination for inequality of pivots may be determined by the same method given for field transit, but it is at best an uncertain quantity, and should be eliminated as nearly as possible from the mean result. If in reversing the telescope the pivots be retained in their respective wyes for both positions, the pivot correction will have opposite signs before and after reversal, and if an equal number of observations are made for each position the mean will not be effected by this error.

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* Chauvenet's Astronomy, Vol. II., p. 53.

The value of the divisions of the striding level may be determined by same method given for transit level, and should be tested for different temperatures, as the difference of the expansions of the metal and glass sometimes produces a strain on the tube that may materially alter its curvature.

The English practice of setting level tubes with plaster of paris is especially faulty, as it is not an uncommon occurrence for tubes thus set to change value of division over 100 per cent. from ordinary variations of temperature.

The usual arrangement for illuminating a telescope by means of a lamp attached to one of the wyes is also liable to produce discrepant results, due to the effect of the heat from the lamp on the axis, wyes and level. For precise work the lamps should be placed on supports, at least one foot distant from the instrument, and arranged with screens for shutting off the heat.

The results from observations obtained with lamp attached to the instrument usually show discrepancies that have the appearance of being due to a change of collimation, but in reality are the effect of the level readings not indicating the true inclination of the rotation axis.

The collimation error should be made as small as possible by adjusting on a distant target, and then eliminated from the mean result by making an equal number of observations before and after each reversal. The amount of collimation may be measured by making readings to the azimuth signal for both positions of the telescope, and will be an excellent check on the results obtained from the star observations.

Diurnal aberration causes the stars to appear to the eastward of their true position by an amount equal to $\frac{0''.308 \cos. A \cos. \varphi}{\sin. z}$.

This correction is always additive to the azimuth of a line when reckoned from the south through 360°.

The post on which the instrument is to be mounted for making the observations should be set as firmly as possible, and have an observatory placed over it to protect the instrument and lights from the wind.

An ordinary coal-oil lantern arranged in a box so that the light may shine through a slit about one-third of an inch wide, and placed at the opposite end of the line of which the azimuth is to be determined, makes an excellent signal for reference of the star observations. This

signal should be at least a mile distant, in order that it may be observed with stellar focus of telescope without apparent parallax.

In commencing a series of observations the telescope may be placed in the meridian in precisely the same manner as field transit, but in determining the error of chronometer a slightly different method may be adopted with advantage. By reading the horizontal circle for position of telescope, when the time observations are made, and referring it to direction of azimuth signal, the azimuth constant for reduction of the time observation may be determined by adding the angle thus obtained to the azimuth of the signal deduced from the observations made on circumpolar stars.

The mean chronometer error obtained from the transits of ten stars will be sufficiently accurate for reducing the azimuth observations, but if the chronometer or watch used does not have a regular rate, part of the observations for time should be made at the beginning and part at the close of the night's work.

The hour angles and azimuths at elongation of the stars to be observed should be computed from formulas 16 and 17 previous to commencing work, to enable the observer to set the telescope so as to bring the stars into the field of view.

The series of observations should be commenced about twenty minutes previous to the time of elongation of star by making careful reading to the azimuth signal, after which the instrument should be turned in azimuth till the telescope is in plane of star and firmly clamped. If the telescope be then moved in altitude until vertical circle is at the reading equal to the latitude of place, the star will be in the field of view.

The observer should then bisect the star with collimated thread of reticule, and by counting the beats of the chronometer note the exact second at which the bisection is made. The micrometers and striding level should then be read as quickly as possible, and the level reversed on the pivots of telescope, and while the bubble is settling another bisection of the star should be made in a similar manner, and the corresponding readings of micrometers and striding level noted. A second reading to the signal will complete the set, and then the telescope should be reversed, the circle turned forward, and the next set of the series observed in the same manner as the preceding.

Let d = the value of one division of the level scale, e, w and e', w' the respective readings of east and west end of striding level before and after

reversal on the pivots, and b = the correction to azimuth for inclination of axis, then $b = \frac{d}{4}\left[(w+w')-(e+e')\right]\frac{\cos. z}{\cos. \varphi}$ (17).

This correction for inclination of axis must be subtracted from the result for azimuth of the signal, when the sum of the readings of west end of scale are the greater.

The mean of four sets of observations with two reversals, if carefully made, will usually give a result for azimuth with a probable error of less than 01″.0.

For precise work, the four standard circumpolar stars given in "American Ephemeris" are usually observed on four different nights, and for secondary work, two of these stars at different elongations are observed on two different nights. The azimuth of meridian lines for State and county surveys are seldom required closer than to the nearest 05″.0, and as an error of one second of time in an observation on Polaris produces only 00″.3 error in the computed azimuth, it is evident that if the chronometer error be determined from the transits of four or five time stars with an ordinary theodolite, then the results for azimuth from observations on Polaris at any hour angle will not be materially affected by the error in the time determination. A convenient length of meridian line for State surveys and for testing the deviation of the magnetic needle is about one mile, the ends of which should be marked with permanent reference stones.

A good form of mark for this purpose consists of a flat stone about 1½ feet square, with a copper bolt leaded in the upper surface. By setting the reference stone about 4 feet deep, with an earthern tile placed above it, there will be but little liability of the mark being disturbed, while the tile enables the engineer to centre an instrument or target accurately over the bolt.

In establishing a meridian line of this kind, if the theodolite be placed over one of the reference points, and the signal light approximately in the direction of the line, the second point may be determined by laying off an offset from the line equal to the distance between reference points, multiplied by the tangent of the observed azimuth.

The observed transits of three equatorial stars across the collimated thread for direct and reversed positions of the telescope of theodolite will give the error of watch with sufficient accuracy, and the mean of two

series of ten readings each between the signal and Polaris should give the azimuth of the line within the required limits of precision.

To establish a meridian line accurately enough for testing the deviation of the magnetic needle is so simple a matter that it should be required by law of every county surveyor. A record of the magnetic variation could thus be obtained for a great number of localities, and reliable magnetic charts formed for the whole country.

Measurement of Angles.

The accuracy with which angles can be measured depends on the precision of the theodolite used and the rapidity with which the observations are made, and since the error in reading a circle with micrometers is less than the error of pointing, it is evident that angles measured with a micrometer theodolite, with the same conditions, will be much more accurate than results obtained by repetitions with a vernier instrument.

The principal source of error in pointing is due to the atmospheric disturbance along the lines of sight, and can only be eliminated by making the observations under as many different conditions as possible.

In the adjustment of a system of triangulation through New York State a few years since, it was found that for a large section of the work angles similarly situated in triangles received corrections having like signs. From an examination of the field notes, it was found that over four-fifths of the measurements were made while a cool north wind was blowing at nearly right angles to the direction of the system, and as one line of stations was on much higher land than the other, the lines of sight were along inclined surfaces, crossed by currents of air of unequal densities. Lateral refraction was therefore produced, and where the conditions were the same for angles similarly situated, errors having like signs were liable to occur. The remeasurement of several of these angles under different conditions substantially confirmed this conclusion.

In order to see over intervening obstacles on lines of primary triangulation, it is usually necessary to make the observations on stations of 10 feet to 100 feet in height.

These stations are built in two parts—the inner part consisting of a tripod of heavy timber, well counterbraced, for supporting the instrument, and the outer part a similar frame, built independent of the tripod,

for supporting the platform on which the observers stand while making measurements.

The action of the heat from the sun as the different sides of the tripod are successively illuminated during the day gives the station a slight twisting motion, which is usually quite regular during the day, but changing to opposite directions after sundown, and continuing to turn backwards until the station regains its normal state.

Assuming that the twist of the station is regular, if an angle be read in both positive and negative directions in equal times, one result will be as much too large as the other is too small.

Hence, where precision is required, readings must be made as quickly as possible, in nearly equal times, and with telescope moved in opposite directions for each alternate measurement.

The mean of a positive and negative measurement of an angle is usually termed a set, sixteen of which, if measured with a good theodolite and under different atmospheric conditions, should give a mean result of sufficient accuracy for primary triangulation.

The method adopted by most engineers for measuring angles with a non-repeating theodolite, is to read consecutively to each station in sight, and closing the series by sighting to the initial station. A second series is then read in the opposite order of stations, completing the set.

The difference of the readings to the consecutive stations gives the respective angles, the sum of which should equal 360°. This condition, however, may lead to erroneous results, for, unless the difference of the sum of the angles from 360° be due to a regular twisting of the tripod and instrument, it should affect only those angles during the measurement of which some change took place in the position of the circle.

If, however, each angle be read separately, the closing condition will add greatly to the accuracy of the work.

This requires more time than is usually advisable to spend on the work, and when a number of angles are to be measured at a station, a convenient method is to measure them in sets of two or three angles each. This allows the observer to read over the more difficult lines at those times when the conditions are the most favorable, which cannot always be done when all the stations are included in the same series.

After each series of readings on an angle or set of angles, the circle should be turned forward an amount equal to the angular distance be-

tween microscopes, divided by the number of series it is proposed to observe.

When the micrometer readings on the test space indicate that a correction for run is required, much labor may be saved by computing the correction for the mean angle instead of for each individual reading. Let $\pm r$ = the correction for run for one revolution of micrometer, and $\pm \triangle'$, $\triangle'' \pm$, etc., the difference on the micrometer scale of the two readings for each measurement of the angle, then for the mean of n observations the correction will be $\frac{\Sigma \triangle}{n} r$.

The instrument and signal, when not placed directly over the geodetic points, must be so located that the corrections to the angles due to their eccentric positions may be computed. If a small theodolite be placed at a short distance from the station in line with some other station, and sighted alternately to the instrument and to a plumb line over the geodetic point, the ordinate of the two with reference to the line may be measured on a scale placed behind the plumb line at right angles to the line of sight. The ordinates of the instrument or signal with reference to the geodetic point for two positions at right angles to each other will enable the observer to compute the correction for any direction.

Accidental errors of observation are as liable to be plus as minus, and, unless large, usually have but little effect on the mean result, but if from any cause an error having a constant sign be made at several of the stations of the system, the effect on the comparison of bases may be such as to render the work apparently worthless.

Errors of this class are usually due to the kind of signal used.

In triangulation, where the lines are from one to five miles long, it is customary with many engineers to use cylindrical or octagonal targets. Hundreds of miles of triangulation have been measured in this way in the surveys made in the United States, and in many cases where accurately measured bases have been connected by such triangulation the computed and measured lengths have not checked within limits that would do credit to ordinary chaining.

In most cases the closure of triangles has been good enough for first-class work, but the error due to phase, although smaller for any individual angle than the accidental errors, is cumulative, and, in a long system of triangulation, may become so large as to render the work worthless.

It may be easily seen, by an examination of a system of triangles, that when the direction of the system is the same as that of average illumination during the work, the errors due to phase in angles opposite the transverse sides in any triangle will have opposite signs, and angles similarly situated in different triangles will have errors with like signs. Each line, therefore, used as a base for computation of triangle sides will be in error by the sum of all the errors due phase in the preceding triangles. If the system is at right angles to the average direction of illumination the effect of the errors on the angles opposite to the transverse sides will be nearly eliminated, but the sum angles between the side lines will be too large on one side of the system and too small on the other, causing the computed azimuths of the sides to curve toward the point of illumination.

Since the average direction of illumination during work usually makes an oblique angle with the direction of the system, the effect of phase will have a tendency to produce errors in both the lengths and azimuths of the lines.

For fixed targets black and white are more readily seen than other colors, but on long lines these colors are often indistinct unless well illuminated.

To obviate this difficulty, the following form of target will be found useful. Bend a stiff wire into a rectangle about 4 feet long, and of such width that it will subtend about 5 seconds of arc on line of sight, and cover it with black and white cloth. By fastening the ends of the wire in a board, the target may be fixed in position over the geodetic point, and may be seen with equal distinctness with the illumination from any direction.

Small targets of this description are especially well adapted for use on secondary triangulation.

On lines exceeding 20 miles in length the heliotrope is generally a more economical form of signal than targets, on account of the distinctness with which it may be seen even through very hazy atmosphere.

A good form of heliotrope for field use consists of a plate mirror about 3 inches square, mounted in gimbals so as to have a horizontal and vertical movement about its centre, and set so that the centre of mirror and the centre of a small aperture in a screen about 20 feet distant are in line between the two stations. When the heliotrope is turned so the rays from the sun reflected from the mirror pass through

the aperture, the reflection may be distinctly seen at observing stations.

Since the rays diverge from the mirror at an angle equal to the angular diameter of the sun, it is not difficult to maintain the mirror in such position as will give a steady signal.

The rays passing through an aperture an inch square will be bright enough for observing on lines 20 to 30 miles long.

The use of this form of heliotrope does not require skilled labor, and will give equally as good results as the more complicated forms of heliotropes, which require an engineer to keep them in adjustment.

The heliotrope is also of great utility for communicating between observing parties when telegraph lines are not convenient.

By making a quick flash across the aperture of screen correspond to a dot, and a flash of a little longer duration to a dash of the Morse telegraph alphabet, the heliotrope becomes an easy method of communication.

Measurement of Bases.

The principal difficulty in making an accurate measurement of a base line arises from the uncertainty in the length of the measuring apparatus at the time used.

Two forms of apparatus, the compensating and the metallic thermometer, are now generally used in Europe and America for measuring base lines, both of which are open to serious objections.

The compensating apparatus consists of a brass and an iron bar firmly fastened together at the rear end of apparatus, and supported on rollers in a metallic tube, so that the bars are free to move parallel to each other from unequal expansions.

The free ends of the bars are connected by a compensating lever in such a manner that the unequal expansions of the bars retain a point on the lever at a constant distance from point of contact at the rear end of the tube for equal temperature of the two bars.

The two metals, brass and iron, do not take on and radiate heat with equal rapidity for changing temperatures, and, consequently, in practice, the bars will seldom be at the same temperature, and the distance between contacts will be a varying quantity.

The metallic thermometer apparatus consists of a steel and a zinc bar fastened together at their centres and supported on rollers in a metallic

tube, so that the ends of the bars are free to move parallel to the axis of the tube.

The upper surfaces of the bars near their ends are cut away to the neutral plane, and are marked with graduations. The distance between the graduations at the ends of the steel bar constitutes the length of the standard, and the amount that this distance changes with reference to the corresponding graduations on the zinc bar, due to the unequal expansions of the two metals, theoretically gives the change of temperature that the bars undergo, but practically does not.

Recent experiments have shown that if the length of a zinc bar is absolutely known at a certain temperature, and that if the temperature of the bar be raised several degrees and afterwards lowered to the original temperature, a set will take place and the length of the bar will be considerably greater than when first derermined. If the temperature be lowered instead of raised, the set will take place in the opposite direction.

The amount of this set at any given time will depend on the last change of temperature that has taken place in the bar, and consequently the temperature of the apparatus derived from the difference in the lengths of the zinc and steel bars may be greatly in error.

A full description, with methods of measuring with these two kinds of base apparatus, may be found in Clark's Geodesy, Coast Survey Reports for 1854 and 1880, and Professional Paper No. 24 of United States Engineer Corps.

If, instead of undertaking to determine the temperature that a bar will assume when used in measuring a base line, the bar be placed in a receptacle filled with melting ice, there is no apparent reason why it may not be maintained at a constant temperature during the entire measurement of a base line, and which may be reproduced at any time when needed in making comparisons with other standards.*

The only inconvenience of this method would be due to the extra weight of the broken ice surrounding the measuring bar. This, however, would not affect the accuracy of the measurements, as the bar could be lined up on its bearings in the receptacle, for a strain due to the weight of ice required, and if the graduations marking the length of the standard are made on the neutral plane of the bar, any slight changes

* This method was originally suggested by Mr. E. S. Wheeler, but, I think, has never been practically tested.

of flexure due to the varying weight of the melting ice would not produce a measurable error in the length of the standard.

An apparatus of this description could probably be used more conveniently as a line measure than otherwise.

The measuring tube having been placed on trestles in line of base, with its rear end over the reference stone marking end of the line, a microscope may be mounted on a trestle at the forward end of the tube, and so adjusted that the micrometer thread will coincide with the graduation marking the length of the standard. The tube should then be carried forward and placed in line with its rear graduation under the micrometer thread, and the forward end marked with another microscope as before.

The inclination of each tube should be read by means of a sector and level, attached at one side of the apparatus.

The simplicity and ease with which observations may be made with an apparatus of this kind, and the fact that no errors could possibly arise from erroneous temperatures, are strong arguments in its favor.

For State and county surveys and secondary triangulation, a steel tape 300 feet to 500 feet long, if properly used, furnishes an easy and accurate means of measuring bases.

The principal uncertainty in the length of a base measured with a steel tape is due to the error in determining the mean temperature of the tape during the measurement.

The time required to measure a base after the line is properly staked out, only amounts to a few hours, and the contacts can be made as readily by means of lamps at night as by daylight.

If small thermometers wound with steel wire, and attached to the tape, are carefully read for each measurement, and the observations made only on cloudy days or during the night, the mean temperature obtained will be very nearly the true temperature of the tape. The coefficient of expansion for steel tapes is 0.0000069, and consequently an error of one degree in the mean temperature would produce an error in the length of base of less than half an inch to the mile.

The modulus of elasticity has been found for steel tapes to be 27400000., and as the cross section of an ordinary tape is about 0.002 square inches, a change of tension of 1 pound in measuring would produce a difference of about 1 inch to the mile in the measured length.

In preparing a base line for measurement stakes should be set at intervals of 25 feet, with nails driven in the sides, at an even grade for supporting the tape, and the difference of elevation of the nails corresponding to the end of each tape length determined by means of a line of levels.

Small wire hooks, about 6 inches in length, are attached to the tape every 25 feet, and when placed on the nails in the base stakes support the tape at an even grade, and allow it free motion when stretched in position for marking contacts.

In measuring, the tape is placed on its supporting hooks, with the rear end over the end of base line and its forward end over a stake driven firmly in the ground, so that its top is on the same level as tape. The tape is then given the tension adopted for its use (usually 10 pounds), the attached thermometers read, and the position of forward end marked with a cut in the head of a tack driven in the stake under the graduation mark.

The length of the tape may be determined by measuring a line of known length in exactly the same manner that it is proposed to use the tape in measuring other lines. The large number of base lines that have been measured in various parts of the country by the United States Engineer Corps and Coast Survey, makes it an easy matter for an engineer to determine the length of a tape measure without having to resort to comparisons with standards.

With proper care, base lines may be measured with a steel tape as accurately as with the secondary base apparatus usually used for this purpose.

Reconnoissance.

The locating of stations for a good system of triangulation requires more experience and better judgment than any other branch of geodetic work.

No general directions can be given that will be applicable to all cases, except that the engineer must be familiar with all the principal topographical features of the country to be traversed by the triangulation, and avail himself of every possible means to determine the heights that will be required for stations at the different points, in order that they may be intervisible.

The stations should be so located, if possible, that the angles of the principal chain of triangles will all exceed 30°.

In secondary triangulation the economy of the work will depend to a large extent on the skill with which the reconnoissance is made.

A single station improperly located may occasion several days' labor for the party to clear the lines of sight, and thus, from delay and damage to property, materially increase the expense of the work.

Secondary systems of triangulation are generally located along some valley or river, and it is possible in most cases to so arrange the positions of stations that the lines of sight on one side or the other will be clear, and thus enable the observer to determine the directions of the lines which need clearing, with sufficient accuracy that only narrow sight lines need be cut.

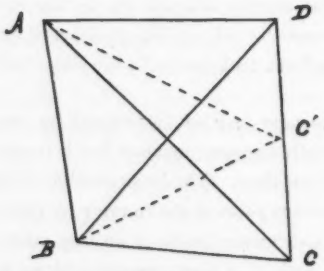

Fig. 3.

Let *A*, *B*, *C*, *D*, be four stations of such a system, and suppose the line *A*, *B*, obstructed by timber. Let flags be put up at the respective stations, and with a sextant or small theodolite read the angles at *C*, between *B*, *A* and *D*, and at *D*, between *A*, *B* and *C*.

Assuming the length of the side *C*, *D*, to be unity, the relative length of the sides *A*, *D*, and *B*, *D*, may be computed from the triangles *A*, *C*, *D*, and *B*, *C*, *D*, which, with the angle at *D* between *A* and *B*, gives two sides and included angle in the triangle *A*, *B*, *D*, with which to compute the relative direction of the side *A*, *B*.

It sometimes so happens that the stations are not intervisible over both the diagonals, in which case a temporary station, *C'*, can generally be so located that the points *A*, *B* and *C* will be visible, and the necessary angles measured for locating the direction of *A*, *B*.

The use of this principle will in many cases increase the progress of the work 25 per cent.